Counting Principle-4

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Q:→ There are 15 points in a plane, no three of which in the same straight line expecting 4, which are collinear. Find the number of
i) straight lines ii) triangles
formed by joining them.

sol:→
(i) We know that join of any two points give a line

$\therefore$ No. of lines got from 15 points $= {}^{15}C_2$

$$= \frac{\lfloor 15}{\lfloor 2 \lfloor 13}$$

$$= \frac{15 \times 14}{2} = 105$$

Lines got from 4 points $= {}^{4}C_2 = \frac{4 \times 3}{2} = 6$

4 collinear pts gives one straigh line.

$\therefore$ Reqd no. of st. lines $= 105 - 6 + 1 = 100$

(ii) We know that any three non collinear points give a triangle.

$\therefore$ No. of triangles gat from 15 points $= {}^{15}C_3$

$$= \frac{\lfloor 15}{\lfloor 3 \lfloor 12}$$

$$= \frac{15 \times 14 \times 13}{3 \times 2}$$

$$= 455$$

Triangles got from 4 points $= {}^4C_3 = {}^4C_1 = 4$

$\therefore$ no. of triangles lost due to 4 collinear pts $= 4$

$\therefore$ Reqd. no of triangles $= 455 - 4 = 451$

Q:→ The number of diagonals of polygon is 20. Find the no. of its sides.

sol:→ Let no. of sides of polygon $= n$

$\therefore$ No. of points $= n$

No. of lines $= {}^nC_2 = \frac{n(n-1)}{2}$

$\therefore$ No. of diagonals $= \frac{n(n-1)}{2} - n$

$$20 = \frac{n(n-1) - 2n}{2}$$

$\Rightarrow n^2 - n - 2n = 40$

$\Rightarrow n^2 - 3n - 40 = 0$

$\Rightarrow (n-8)(n+5) = 0$

$\Rightarrow n = 8, \cancel{-5}$

$\therefore$ No. of sides of the polygon $= n = 8$ Ans

Multi set :→

A collection of objects in which objects are allowed to be repeated is called a multi set e.g $S = \{a, a, a, b, b, c\} = \{3 \times a, 2 \times b, 1 \times c\}$

In general, $S = \{r_1 \times a_1, r_2 \times a_2, \dots, r_k \times a_k\}$ is a multi set in which a_1 occurs r_1 times, a_2 occurs r_2 times and so on, a_k occur r_k times.

r - Permutation :→

By an r-permutations, we mean an ordered arrangement of r elements from S. If $|S| = n$, then n - permutation of S is just called a permutation of S.

1) If S is a multiset with infinite repetitions then the number of r permutations of S is k^r where k is the no. of distinct objects in S.

Q:→ Find the number of ternary numbers with atmost 4 digits.

sol:→ The number of ternary numbers with atmost 4 digits.

$= 4$-permutation with 3 objects $(0, 1, 2)$

$= 3^4$

$= 81$

0, 1, 2

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$r \le n$
r - perm. of n distinct

2) If $S = \{n_1 \times a_1, n_2 \times a_2, \dots, n_k \times a_k\}$

i.e $|S| = n_1 + n_2 + \dots + n_k = n$

Then the no. of permutations of S i.e n permutations is $\dfrac{\lfloor n}{\lfloor n_1 \lfloor n_2 \dots \lfloor n_k}$

Q:→ Find the number of ways 23 books can be given to 5 students so that 2 students will have 4 books and other three students have 5 books each.

sol:→ $S = \{2 \times 4, 3 \times 5\}$ Gp1 2, Gp2 3 students

$|S| = 2 + 3 = 5$

$\therefore$ Reqd no. of ways $= {}^5C_2 \times \frac{\underline{|23}}{\underline{|4}\,\underline{|4}\,\underline{|5}\,\underline{|5}\,\underline{|5}}$

Q:→ In how many ways can 7 women and 3 men be arranged in a row if 3 men must be together?

sol:→ Regarding 3 men as a group of one.

$S = \{W_1, W_2, W_3, W_4, W_5, W_6, W_7, 3 \times M\}$

Total no. of arrangement $= \underline{|8} \times \underline{|3}$ Ans

3) The no. of circular permutations of n objects $= \underline{|n-1}$

Ex:→ Find the no. of non-negative integers $< 10^9$ which contain digit 1.

Ans $10^9 - 9^9$